Tips from the Writers of the CCSSM

Alicia Stoltenberg
Perneet Sood
Background

- Student Achievement Partners
  - Co-founded by: Jason Zimba, David Coleman, Sue Pimentel
  - Advisors: Phil Daro, Bill McCallum

- www.achievethecore.org

Source about founders/advisors: http://www.achievethecore.org/student-achievement-partners
Wording from the CCSSM
SBAC and PARCC both had vendors at this workshop

SAP is aiming to maintain the integrity of the CCSSM through testing companies
Just in case you were wondering...

The Three Shifts in Mathematics

- **Focus** strongly where the standards focus
- **Coherence:** Think across grades and link to major topics within grades
- **Rigor:** In major topics, pursue with equal intensity:
  - Conceptual understanding
  - Procedural skill and fluency
  - Application

Rigor does not necessarily equate to “more difficult”

The three components of Rigor need to be pursued in balance (equal intensity)
Shift One: **Focus**
strongly where the Standards focus

- The CCSSM significantly narrow the scope of content and deepen how time and energy are spent in the mathematics.

- Focus deeply only on what is emphasized in the standards, so that students gain strong foundations.
“Shopping cart” approach
• Take a little bit here and there from the aisles, throw it into the cart. Keep moving on.
Organization from Illustrative Mathematics site
Focus on the Major Work of the Grade

- Two levels of focus
  - What’s in/What’s out
  - The shape of the content that is in

“What’s in/out” – example of out: pattern work in 3-5

Major work of the grade is written out on the first page of each grade level– The “critical areas”
Mathematics Working Session Kickoff
CCSSM Overview

Shift Two: Coherence

Think across grades, and link to major topics within grades

- Carefully connect the mathematics within and across grades so that students can build new understanding onto foundations built in previous years.
- Each year, we expect solid conceptual understanding of core content and build on it. Each standard is not a new event, but an extension of previous learning.

Emphasis on the last part--- each standard is an extension of previous learning— NOT A NEW EVENT!
Mathematics Working Session Kickoff

CCSSM Overview

Coherence: Think across grades

Fraction example:
“The coherence and sequential nature of mathematics dictate the foundational skills that are necessary for the learning of algebra. The most important foundational skill not presently developed appears to be proficiency with fractions (including decimals, percents, and negative fractions). The teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected.”

Explicit connections are even spelled out in the standards.
Explicit connections are even spelled out in the standards.

3.MD.C – “C” is for the 3rd cluster. A naming convention of the clusters as A, B, C, etc. has been informally established/reinforced by several groups.
Depth of standards

Mathematics Working Session Kickoff
CCSSM Overview

Shift Three: **Rigor**

Equal intensity in conceptual understanding, procedural skill/fluency, and application

- The CCSSM require a **balance** of:
  - Solid conceptual understanding
  - Procedural skill and fluency
  - Application of skills in problem solving situations

- This requires equal intensity in time, activities, and resources in pursuit of all three.
(a) Solid Conceptual Understanding

- Standards require that students know more than “how to get the answer”.
- Instructional and assessment tasks must provide access to concepts from a number of perspectives to show deep understanding.
- Math is more than a set of mnemonics or discrete procedures.
- Conceptual understanding supports the other aspects of rigor (fluency and application).

<table>
<thead>
<tr>
<th>Arithmetic with Polynomials and Rational Expressions</th>
<th>A-APR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perform arithmetic operations on polynomials</td>
<td></td>
</tr>
</tbody>
</table>

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Tests may test procedural skill

Intent of CCSSM is to focus on deeper understanding of fewer topics
- Not memorization absent understanding
- Many students will need practice in fluency
- Fluency as in a foreign language: use/perform math fluently as you would speak a foreign language fluently

### Mathematics Working Session Kickoff

#### CCSSM Overview

(b) Fluency

- The standards require *speed* and *accuracy* in calculation.
- Fluency is called for explicitly in certain standards.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluency</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
</tr>
<tr>
<td>2</td>
<td>2.OA.2</td>
<td>Add/subtract within 20 (know single-digit sums from memory)</td>
</tr>
<tr>
<td></td>
<td>2.NBT.5</td>
<td>Add/subtract within 100</td>
</tr>
<tr>
<td>3</td>
<td>3.OA.7</td>
<td>Multiply/divide within 100 (know single-digit products from memory)</td>
</tr>
<tr>
<td></td>
<td>3.NBT.2</td>
<td>Add/subtract within 1000</td>
</tr>
<tr>
<td>4</td>
<td>4.NBT.4</td>
<td>Add/subtract within 1,000,000</td>
</tr>
<tr>
<td>5</td>
<td>5.NBT.5</td>
<td>Multi-digit multiplication</td>
</tr>
<tr>
<td>6</td>
<td>6.NS.2,3</td>
<td>Multi-digit division</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Multi-digit decimal operations</td>
</tr>
</tbody>
</table>
Mathematics Working Session Kickoff

CCSSM Overview

(c) Application

- The standards require that students use appropriate concepts and procedures for application even when not prompted to do so.
- Provide opportunities at all grade levels for students to apply math concepts in “real world” situations, recognizing this means different things in K-5, 6-8, and HS.
It starts with **Focus**

- U.S. curriculum is thought of as ‘a mile wide and an inch deep’.
- Focus is necessary in order to achieve the rigor set forth in the standards.
- More in-depth mastery of a smaller set of things pays off.

For test developers, “focus” means a test blueprint
Elementary level

Non-example: The wording of the standard says “generate” not “complete” or “extend”. Standard explicitly says, “that follows a given rule”.

Mathematical practices
Elementary level

Original and improved items from PARCC

Focus on groups of objects in the second “better” example. Also, uses less familiar numbers that the student might not as likely “count out”

Mathematical practices
Features of CCSSM and Implications for Assessment
Assessing Individual Content Standards or Parts of Standards
Examples and Non-Examples

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

A circular field with diameter 100 meters is planted with soybeans. How many square meters are planted with soybeans? \( A = \pi r^2, \pi \approx 3.14 \)

Middle level

Non-example: Wording of standard says “know the formulas”

Mathematical practices
Middle level

Non-example: There is no language in the standard about simplifying, even though the example shows simplification of a numerical expression.

Zimba expressed that the example (in 8.EE.1) doesn’t do the standard justice here.

Mathematical practices
Middle level

Not great, but better than the last example: Better exponents for 8th graders, but still just applying the rules (not much understanding shown)

Focus should NOT be working with integers, but knowing how to work with expressions

Classroom focus can use all of the skills known, but summative test will hone in on specific skills

Mathematical practices
Current drafts of progressions are found on the arizona.edu website.

You can also do a Google search for “math progressions” and it’s near the beginning (if not the 1st!) result

Across grade levels
- pre-assessments to determine if material was learned (CETE formative tools as an option)

Grain size of progressions – smaller within grade, larger across grades
Pivot points -> sources of motion

This is an illustration of how multiplication (starting in grade 2) sets the foundation for concepts that will be developed in later grades.

Teacher awareness of cross-grade level knowledge (neighboring grades)
How do you assess the quality of a task? Potential guidelines for assessing materials

Zimba: No amount of specification specifies anything.
Features of CCSSM and Implications for Assessment
Assessing Individual Content Standards or Parts of Standards
Draft Quality Criteria and Table Activities

Example Tasks

Elementary
Middle
Secondary
Elementary: Example task #1

Alisa had $\frac{1}{2}$ of a liter of juice. She drank $\frac{3}{4}$ of the juice. How many liters of juice did she drink?

Elementary Level

5.NF.6

Real world problem solved with multiplication of fractions (how real world? Not sure)

Method for solving is left open to the student. Examples follow.

Mathematical practices
Elementary Level

Fraction model solution
Solution: Using a Number Line

First plot a point at $\frac{1}{2}$, that represents the amount of juice in the bottle.

Divide the segment between 0 and $\frac{1}{2}$ into four equal lengths so that we can plot $\frac{1}{4}$ of $\frac{1}{2}$, Also divide the segment between $\frac{1}{2}$ and 1 into four equal lengths so we can see that the segment between 0 and 1 is divided into 8 equal lengths. Labeling the segments in eighths, we can see $\frac{3}{8}$ of $\frac{1}{2}$ is $\frac{3}{8}$.  

Because each half liter has been divided into 4 equal pieces, the whole liter has been divided into 8 equal pieces. Since what Alice drank is represented by 3 of these pieces, she drank $\frac{3}{8}$ liters of juice.

Source: www.illustrativemathematics.org
Features of CCSSM and Implications for Assessment
Assessing Individual Content Standards or Parts of Standards
Draft Quality Criteria and Table Activities

Elementary: Example task #2

A. Helen raised $12 for the food bank last year and she raised 6 times as much money this year. How much money did she raise this year?

B. Sandra raised $15 for the PTA and Nita raised $45. How many times as much money did Nita raise as compared to Sandra?

Source: www.illustrativemathematics.org

Elementary Level

4.OA.2

Can’t decide 3rd or 4th?
3rd grade: equal groups; 4th grade: times as much

Method for solving left open to student

Improve: something more concrete than money

Mathematical practices
Features of CCSSM and Implications for Assessment
Assessing Individual Content Standards or Parts of Standards
Draft Quality Criteria and Table Activities

Solution: Tape diagram
a. She raised six times as much money (as shown in the diagram) so she raised $6 \times 12 = 72$.

Money she raised last year: $\text{\underline{\$12}}$
Money she raised this year: $\text{\underline{\$12}} \text{ \underline{\$12}} \text{ \underline{\$12}} \text{ \underline{\$12}}$ ?

Helen raised $\text{\$72}$ this year:
b. $7 \times 15 = 45$ is equivalent to $45 : 15 = ?$

Money Sandra raised: $\text{\underline{\$15}}$
Money Nita raised: $\text{\underline{\$45}}$

Nita raised 3 times as much as Sandra.

Source: www.illustrativemathematics.org
Features of CCSSM and Implications for Assessment
Assessing Individual Content Standards or Parts of Standards
Draft Quality Criteria and Table Activities

Solution: Writing multiplication equations for division problems
a. Helen raised $6 \times \$12$ this year, so she raised $\$72$ this year.
b. This is a “Number of Groups Unknown” problem. We can represent the question as

\[ ? \times 15 = 45 \]

or

\[ 45 \div 15 = ? \]

So Nita raised 3 times as much money as Sandra.

Source: www.illustrativemathematics.org
Middle: Example task #1

The students in Mr. Nolan’s class are writing expressions for the perimeter of a rectangle of side length $\ell$ and width $w$. After they share their answers, the following expressions are on the board:

- Sam: $2(\ell + w)$
- Joanna: $\ell + w + \ell + w$
- Kiyo: $2\ell + w$
- Erica: $2w + 2\ell$

Which of the expressions are correct and how might the students have been thinking about finding the perimeter of the rectangle?

Source: www.illustrativemathematics.org

Middle Level

6.EE.4

If there were no context, maybe 6.EE.2a. More likely at the cluster level.

Method for solving left open to student

Improve: Ask about the thoughts of Kiyo (wrong answer), give a rubric for scoring

Mathematical practices: 1, 3, 7
Middle Level
Middle Level

8.NS.1

Improve: Less questions, more explanations.
Have fewer and compare sizes (would change alignment to 8.NS.2)

It would be interesting to compare the answers of C and D.

Mathematical practices
Secondary Level

A-SSE.1, A-SSE.2

Item writing at the finest grain in HS is tricky, as the HS standards don’t stand up well in isolation.

Mathematical practices
Secondary: Example task #2

Given a function $f$, is the statement

$$f(x + h) = f(x) + f(h)$$

true for any two numbers $x$ and $h$? If so, prove it. If not, find a function for which the statement is true and a function for which the statement is false.

Source: www.illustrativemathematics.org
Secondary Level
SBAC Sample Item - How do these items look different from what we’ve been using for the last ~10 years?

3.NF.3a

Visit their website for more...
SBAC Sample Item - How do these items look different from what we’ve been using for the last ~10 years?

6.EE.4

Visit their website for more...
SBAC Sample Item - How do these items look different from what we’ve been using for the last ~10 years?

HS – N-RN.1, 8.NS.2

Non-calculator item.

Visit their website for more...
SBAC Sample Item - How do these items look different from what we’ve been using for the last ~10 years?

HS – A-CED.1, A-REI.3

Visit their website for more...
SBAC Sample Item - How do these items look different from what we’ve been using for the last ~10 years?

HS – 8.EE.6

Visit their website for more...
Q = question (item)
Standards are “buckets”
Some standards have a very procedural focus

Buckets should connect to other buckets (either within or across grades)

Cluster headings provide the coherence needed for context within and across grades.

Zoom out... Just because it fits one standard/sub standard does not mean that it cannot also be another.
There could be one more bucket: Grade 5 tasks

Sometimes the whole is the sum of the parts (writing items @ standard level):
- Have I added to coherence?
- Have I introduced a new nuance that detracts from the coherence?
- New expectations??

More need for a task model to right of the scale.

Score reporting? – breaking it down too much is not helpful

What about the depth of the items? Where is the innovation? The richness?

Tweaking one small thing in an item can change the complexity entirely – such as changing the numbers.

There is value in asking the standard the “opposite” way. Such as “Given A, find B.”
Turn it around. “Given B, find A.”

The cluster, domain, and grade levels of CCSSM all hold meaning. Both SBAC and PARCC are exploring this feature of the standards.
There are a finite set of key opportunities to assess at the cluster, domain, and grade levels.
Such opportunities should be identified carefully and must make obvious educational and mathematical sense within the framework of the standards. For example, the first two clusters in NBT are often important to link; and doing simple word problems to apply developing NBT skills requires some grade-level tasks that blend OA and NBT.

This is not a recommendation to make loose interpretations of the standards or go beyond what is written in the standards. Rather, it is an opportunity to measure plausible and immediate implications of what is written in the standard, without ever slipping into the imposition of additional requirements.
Features of CCSSM and Implications for Assessment
Assessing at All Levels of the Content Hierarchy

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No clear standard alignment

Where in your learning cycle would you use a task like this? What time of year?
Features of CCSSM and Implications for Assessment
Assessing at All Levels of the Content Hierarchy

A second illustrative task at the domain level...

The following problem is plausible as a direct implication of the Grade 4 standards for Number and Operations in Base Ten, but it involves place value understanding as well as elements of place value computation, so it blends the first two clusters in 4.NBT.

| 893,462 |  |
| 840,924 |  |
| 824,595 |  |
| 824,162 |  |
| 810,930 |  |
| 808,879 |  |
| 799,982 |  |
| 778,877 |  |
| 777,852 |  |
| 766,398 |  |

Find two numbers in this table that differ by approximately two thousand.

Source: PARCC Item Development ITN, Appendix F
Features of CCSSM and Implications for Assessment
Assessing at All Levels of the Content Hierarchy

A third illustrative task at the **domain** level...

The following illustration is plausible as a direct implication of the Grade 5 standards for Number and Operations in Base Ten, but involves place value understanding as well as elements of place value computation, so the illustration blends the first two clusters in **5.NBT**.

A bakery made 3,200 cookies and needs to package them in bags for sale. 10 cookies will go in each bag. How many bags will be filled?

Source: PARCC Item Development ITN, Appendix F
A small number of HS standards make sense in isolation (Zimba)

### Features of CCSSM and Implications for Assessment

**Assessing at All Levels of the Content Hierarchy**

An illustrative grade-level task assessing 6.RP.3 and 6.G.1-4...

#### Painting a Barn

Alexis needs to paint the four exterior walls of a large rectangular barn. The length of the barn is 80 feet, the width is 50 feet, and the height is 30 feet. The paint costs $28 per gallon, and each gallon covers 420 square feet. How much will it cost Alexis to paint the barn? Explain your work.

Source: www.illustrativemathematics.org
Fluency is not a part of every standard.

The standards make it very apparent that things to be done “fluently” are mostly pure computation, not real-world situations.

There is some conceptual understanding that goes along with fluency.
### Table Activity: Measuring Fluency

What types of fluency assessment have you seen used successfully? What kinds of new approaches should be tried out?

How can technology help support the assessment of fluency?

How important is speed in fluency?

---

Share as a whole group after small group discussion. Any common themes? Anything we could use at CETE?

Timed vs. Untimed? (speed)
--keeping track of time without “keeping time”
Assessing Conceptual Understanding of a Particular Piece of Pure Mathematics

Why is it important to talk about conceptual understanding?

- As part of the standards’ balance mathematical rigor, many individual standards set explicit expectations for conceptual understanding of key mathematics.

There is a world of difference between a student who can summon a mnemonic device to expand a product such as \((a + b)(x + y)\) and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding \((a + b + c)(x + y)\).

Mathematical understanding and

Source: CCSSM, page 4
“understand” – what one participant called the standards which begin with “Understand”
Assessing Conceptual Understanding of a Particular Piece of Pure Mathematics

What does assessing conceptual understanding look like?

- Questions assessing understanding of a mathematical concept can vary in length and format.
  - They can be briefly stated and briefly answered questions that are simple to answer if a student understands the concept, and difficult to answer if the student does not—or slightly longer tasks that require students to explain.
  - Long tasks probably drift away from the goal of assessing conceptual understanding of a particular mathematical concept as called for in the standards.
Assessing Conceptual Understanding of a Particular Piece of Pure Mathematics

Fill in the blanks:

591: ___ hundreds, ___ tens, ___ ones
706: ___ hundreds, ___ tens, ___ ones
822: ___ hundreds, ___ tens, ___ ones
267: ___ hundreds, ___ tens, ___ ones
193: ___ hundreds, ___ tens, ___ ones

NON-example.

This task does NOT assess conceptual understanding of place value. Once the student figures out the first one, they simple transfer the numbers to the blanks. The blanks are also in the same order as place value.
Assessing Conceptual Understanding of a Particular Piece of Pure Mathematics

Fill in the blanks:

A. 1 hundred + 4 tens = ____
B. 4 tens + 1 hundred = ____
C. 14 tens = 10 tens + ____ tens
   = ____ hundreds + 4 tens
   = ____ ones
D. 90 + 300 + 4 = ____

Better assessment of the concept of place value.

Moving around where students are to fill in the blank does not allow them to simply fill it in as they go.
Assessing Conceptual Understanding of a Particular Piece of Pure Mathematics

Example #1

Write a number that is greater than $\frac{1}{5}$ and less than $\frac{1}{4}$.

43% of adults said there is no such number!!

Handout questions:
How would you code this task to the standards?
Describe why the task does or does not assess conceptual understanding of the pure mathematics in the standard(s) you identified.
What is the role of technology in assessing conceptual understanding?
Assessing Conceptual Understanding of a Particular Piece of Pure Mathematics

Example #2

Check the box if the given expression is a polynomial.

- $(3 - t^2)(4 - t^2)(5 - t^2)$
- $\pi r^2$
- $\frac{2}{3}$
- $\frac{1}{2x}$
- $1 - t^2$
- $\sqrt{2}$

Source: Hypothetical example by Jason Zimba

A-APR.1

Many students view the 2nd as just a formula

Handout questions:
How would you code this task to the standards?
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What is the role of technology in assessing conceptual understanding?
8.NS.1

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Assessing Conceptual Understanding of a Particular Piece of Pure Mathematics

The “two-second rule” is used by a driver who wants to maintain a safe following distance at any speed. A driver must count two seconds from when the car in front of him or her passes a fixed point, such as a tree, until the driver passes the same fixed point. Drivers use this rule to determine the minimum distance to follow a car traveling at the same speed. A diagram representing this distance is shown.

As the speed of the cars increases, the minimum following distance also increases. Explain how the “two-second rule” leads to a greater minimum following distance as the speed of the cars increases. As part of your explanation, include the minimum following distances, in feet, for cars traveling at 30 miles per hour and 60 miles per hour.

Source: www.smarterbalanced.org

SBAC Sample Item 43060

Conceptual understanding of d=rt

Targeting F-BF.1a, F-LE.1b
Assessing Conceptual Understanding of a Particular Piece of Pure Mathematics

Sample Top-Score Response:
The minimum following distance is determined by the formula \( d = rt \), where \( d \) is the minimum following distance, \( r \) is the rate (or speed), and \( t \) is the time. The “two-second rule” says that the time needed between cars traveling at the same speed remains constant at 2 seconds, so as the speed of the cars increases by a certain factor, then the minimum following distance must increase by the same factor. Since the speed of the cars is measured in miles per hour, and the “two-second rule” measures time in seconds, I used the formula shown below to determine the minimum following distance, in feet.

\[
    d = r \cdot \left( \frac{5280}{1} \right) \cdot \left( \frac{1}{3600} \right) \cdot 2
\]

For cars traveling at 30 miles per hour, the minimum following distance is 88 feet. For cars traveling at 60 miles per hour, the minimum following distance is 176 feet.

Source: www.smarterbalanced.org

SBAC Sample Item 43060

Rubric – Sample top score response

Conceptual understanding of \( d=rt \)

Targeting F-BF.1a, F-LE.1b

Level 4, Evaluate
Assessing Conceptual Understanding of a Particular Piece of Pure Mathematics

Some possible rules of thumb for writing conceptual understanding tasks:

- Ratchet down the computational skill required.
- Have a concept in mind when you are writing a task.
- Understand the concept yourself: What are its basic components?
- Consider the “efficiency” of the task. How many points are being generated in how much time? How important is the topic?
- The task could be very easy to solve if concept is understood by the student.
- It is important that the format of these tasks are non-routine (tasks assessing conceptual understanding are good candidates for technology enhancement for this reason).
- Assessing conceptual understanding is about understanding if a student can do the pure mathematics. Does not include “transfer” to application setting.

- Connections – vertical, horizontal, every which way (in grade, across grades, etc)
- Connections between concepts → lots of literature on this (Essential Understandings) – Representations, Recognition
- Understand – multiple levels of understanding; tasks can have content grades and cognitive grades
Practices look differently at different grade levels.

Introduction to practices early—incorporate into item reviews/table activities
- use with items which are good examples of a MP in use
Connecting Content Standards and Mathematical Practices in Assessment

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Source: CCSSM, page 6

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

Source: CCSSM, page 8
There is a potential split at “and”

2\textsuperscript{nd} alternative: loss of solution focus

Final: Persevere! This is when a student can answer the question, “When am I done?”

What happens when you get the answer? Do you reflect? Check?
Design prompts to allow time for sense-making, no immediate jump to operations

Time must be age appropriate – 10-20 minutes is too long for young kids

“nonroutine”

For example: Find a number that rounds up to nearest tenth, and down to nearest hundredth. Simple content, but yet not routine.

What it is not: Algorithm based, such as, what’s the distance between (2,3) & (-4,7)?
Part of the task can be to decide whether or not to use certain tools

Tools OTHER than a calculator: # line, coordinate plane, diagrams, formulas...

"Is this right?" — a MP6 question

Assessing the MPs in a standalone way is not an option. They must be in conjunction with content standards.
Connecting Content Standards and Mathematical Practices in Assessment

MP.5: Use appropriate tools strategically.

Considerations for Assessment Development:

- Problems that are easy to solve by using a mathematical or technological tool, and fairly hard to solve without doing so; no explicit instruction is given to students to use the tool.
- Tools may include: coordinates, diagrams, formulas, conversions or other mathematical knowledge, ruler, spreadsheet, calculators, etc.
- Appropriate tools vary by grade band, grade, even by standard.
- Tasks should assess the strategic use of tools; every time one of the tools is used does not mean that students are engaging in the practice. Tasks might create circumstances for poor use of tools, or misuse of tools, or mistakenly not using tools.

Source: Excerpted from SBAC Content Specifications, PARCC Item Development ITN, Appendix D
Connecting Content Standards and Mathematical Practices in Assessment

MP.7: Look for and make use of structure.

What if the standard had instead said...

MP.7: Work on problems where structure is present.

MP.7: See structure.

MP.7: Look for and make use of structure.

It is best if tasks targeting MP.7 put the structure to some real-world or mathematical purpose.
Connecting Content Standards and Mathematical Practices in Assessment

MP.7: Look for and make use of structure.

Considerations for Assessment Development:

- Mathematical and real-world problems that reward seeing structure in an algebraic expression and using the structure to rewrite it for a purpose
- Problems that assess how aware students are of how concepts link together, and why mathematical procedures work in the way that they do
- Numerical problems that reward or require deferring calculation steps until one sees the overall structure
- Geometric problem solved by analyzing parts of figures in relation to one another, or by introducing auxiliary lines into a figure.

Source: Excerpted from SBAC Content Specifications, PARCC Item Development ITN, Appendix D
Connecting Content Standards and Mathematical Practices in Assessment

MP.8: Look for and express regularity in repeated reasoning.

*What if the standard had instead said...*

**MP.8:** Perform repeated reasoning.

**MP.8:** Express regularity in repeated reasoning.

**MP.8:** Look for an express regularity in repeated reasoning.

*If a task matches the first version, it is a non-example of MP.8. If it matches the second, it is at least a partial example.*
Connecting Content Standards and Mathematical Practices in Assessment

MP.8 and “Patterns”

MP.8 is sometimes associated with “patterns.” However, MP.8 is more subtle than traditional pattern work, and CCSSM calls for a shift overall in the approach to patterns.

“Patterns are a tool, not a topic.” - The use of patterns must always shed light on the topic at hand:

- In K-5, the topic at hand is overwhelmingly arithmetic: thus, patterns can be used to shed light on the addition table, the times table, the properties of operations, the relationship between addition and subtraction or multiplication and division, and the place value system. There is no extending patterns for their own sake, “guessing the rule,” or other proto-function content.
  - The answer might be something like, “multiplying by any number and then dividing by the same number gets you back to where you started.”
- In 6-8, the topic at hand will often be proportional relationships and linear functions. In high school, the topic at hand will often be formal algebra as well as functions (particularly recursive definitions of functions).
  - The answer might be an equation, a function, or a conjecture (not a number).
- In all cases, remember that the pattern is a tool, and MP.8 requires expressing regularity somehow, not just doing something over and over or extending a pattern.

“When I...Then I...” -- Phil Daro

No standard in CCSSM asks students to extend a pattern.

3.OA.1 – Generate a pattern from a rule (NOT extend!)
Connecting Content Standards and Mathematical Practices in Assessment

MP.8: Look for and express regularity in repeated reasoning.

Considerations for Assessment Development:

- Not just engaging in repeated reasoning, but expressing the regularity that emerges from it, and gaining the habit of mind of looking for it.
- Patterns are a tool, not a topic.
- Problems in which a tedious and repetitive calculation can be made shorter by observing regularity in the repeated steps.
- Mathematical problems in which repeated calculations lead to the articulation of a conjecture.
- Modeling problems in which working repetitively with numerical examples leads without prompting to the writing of equations or functions that describe the situation.

Source: Excerpted from SBAC Content Specifications, PARCC Item Development ITN, Appendix D
Connecting Content Standards and Mathematical Practices in Assessment

Assessing MP.3

Both SBAC and PARCC have identified claims related to MP.3

SBAC Claim #3: Communicating Reasoning. Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

- Rigor in reasoning is about the precision and logical progression of an argument: first a student must avoid making false statements, then say precisely what one assumes, and provide the sequence of deductions made on this basis. “Assessments for this claim should use tasks that examine a student’s ability to analyze and provided explanation, to identify flaws, to represent a logical sequence, and to arrive at a correct argument.” - SBAC Content Specifications 3.20.12, pg. 62.

PARCC Sub-Claim C: Expressing Mathematical Reasoning. The student expresses grade/course-level appropriate mathematical reasoning by constructing viable arguments, critiquing the reasoning of others and/or attending to precision when making mathematical statements.

- Tasks that call for written arguments/justifications, critique of reasoning, or precision in mathematical statements (MP.3, MP.6) - PARCC Appendix F
Careful design of prompts. Be specific rather than just say, “Explain.”

HOW do you want them to explain? Draw a picture? Write a few sentences?

Connecting Content Standards and Mathematical Practices in Assessment

MP.8: Look for and express regularity in repeated reasoning.

Considerations for Assessment Development:
- Look for obvious opportunities where content standards are asking for reasoning about key grade-level mathematics.
- Avoid use of the “explain” prompt. Instead, ask student to “use diagrams, words, and/or equations to...”.
- It is much easier to repair a flawed argument than to diagnose the problem with it. The former is likely more appropriate in K-5.
- Problem solving is a form of argument when the solution is to be laid out as a series of logical and well-motivated steps using precise language and terms (see also MP.6)

Source: Excerpted from SBAC Content Specifications, PARCC Item Development ITN, Appendix D

The University of Kansas
Connecting Content Standards and Mathematical Practices in Assessment

Assessing MP.4

Both SBAC and PARCC have identified claims related to MP.4

SBAC Claim #4: Modeling and Data Analysis. Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

- Items “assess student expertise in choosing appropriate content and using it effectively in formulating models of the situations presented and making appropriate inferences from them” (SBAC Content Specifications)

PARCC Sub-Claim D: Modeling/Application. The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course, engaging particularly in the Modeling practice...

- Tasks that involve real-world contexts or scenarios and require the student to apply knowledge and skills articulated in specified standards; engage particularly in the Modeling practice (MP.4); and where helpful make sense of problems and persevere in solving them (MP.1) reason abstractly and quantitatively (MP.2) use appropriate tools strategically (MP.5), look for and make use of structure (MP.7) and/or look for and express regularity in repeated reasoning (MP.8) (PARCC Item Development ITN, Appendix F)
Connecting Content Standards and Mathematical Practices in Assessment

**MP.4: Model with mathematics.**

*General Considerations for Assessment Development:*

- Modeling tasks may be ill-posed or slightly ambiguous: problem formulation is part of the problem; the student is confronted with a contextualized or “real world” situation and must decide which information is relevant, and how to represent it.

- Tasks involve formulating a problem that is tractable using mathematics – that is, formulating a model. This will usually involve making assumptions and simplifications; students will need to select from the data at hand, or estimate data that are missing.

Source: Excerpted from SBAC Content Specifications, PARCC Item Development ITN, Appendix D
The Shell Center is developing some interesting real-world tasks
Connecting Content Standards and Mathematical Practices in Assessment

Additional Resources

If you are looking for more information, be sure to check out the following resources:

- **SBAC Content Specifications**: Assessment Targets for claims 2, 3, and 4

- **PARCC Item Development ITN, Appendix D**: Evidence Statements for sub-claims C and D, discussion about practice-forward tasks

When trying to focus more on the mathematical practices, decrease the level of the content standards.

When trying to focus more on the content standards, decrease emphasis of the practices.
Thank you!

Alicia Stoltenberg
alia.stoltenberg@ku.edu

Perneet Sood
psood@ku.edu
Assessing Standards - Elementary #1

Alisa had $\frac{1}{2}$ of a liter of juice. She drank $\frac{3}{4}$ of the juice. How many liters of juice did she drink?

How would you code this task to the standards?

Describe the degree of alignment, taking into account the context.

Could this task be improved to better align, or improved in other ways?

What questions did you ask while examining the tasks to determine quality and/or alignment?

Source: www.illustrativemathematics.org

5.NF.6 from illustrativemathematics.org
Assessing Standards - Elementary #2

A. Helen raised $12 for the food bank last year and she raised 6 times as much money this year. How much money did she raise this year?

B. Sandra raised $15 for the PTA and Nita raised $45. How many time as much money did Nita raise as compared to Sandra?

How would you code this task to the standards?

Describe the degree of alignment, taking into account the context.

Could this task be improved to better align, or improved in other ways?

What questions did you ask while examining the tasks to determine quality and/or alignment?

Source: www.illustrativemathematics.org

4.OA.2 from illustrativemathematics.org
Assessing Standards - Elementary #3

Leo and Silvia are looking at the following problem:

How does the product of 60 × 225 compare to the product of 60 × 225?

Silvia says she can compare these products without multiplying the numbers out. Explain how she might do this. Draw pictures to illustrate your explanation.

How would you code this task to the standards?

Describe the degree of alignment, taking into account the context.

Could this task be improved to better align, or improved in other ways?

What questions did you ask while examining the tasks to determine quality and/or alignment?

Source: www.illustrativemathematics.org

5.OA.2 from illustrativemathematics.org

MP #7 – Deferring calculation and seeing structure makes the problem simpler.
A recipe for chocolate chip cookies makes 4 dozen cookies and calls for the following ingredients: 
1 1/2 C butter 
1 1/2 C sugar 
2 1/2 C flour 
1 t baking powder 
1/2 t salt 
8 oz chocolate chips 
A. How much of each ingredient is needed for 12 dozen cookies? 
B. How much of each ingredient is needed for 3 dozen cookies?
Assessing Standards - Middle #1

The students in Mr. Nolan’s class are writing expressions for the perimeter of a rectangle of side length $\ell$ and width $w$. After they share their answers, the following expressions are on the board:

- Sam: $2(\ell + w)$
- Joanna: $\ell + w + \ell + w$
- Klio: $2\ell + w$
- Erica: $2w + 2\ell$

Which of the expressions are correct and how might the students have been thinking about finding the perimeter of the rectangle?

How would you code this task to the standards?

Describe the degree of alignment, taking into account the context.

Could this task be improved to better align, or improved in other ways?

What questions did you ask while examining the tasks to determine quality and/or alignment?

Source: www.illustrativemathematics.org

6.EE.4 from illustrativemathematics.org
Assessing Standards - Middle #2

Decide whether each of the following numbers is rational or irrational. Explain how you know.

A. 0.333
B. \( \sqrt{4} \)
C. \( \sqrt{2} = 1.414213... \)
D. 1.414213
E. \( \pi = 3.141592... \)
F. 11
G. \( \frac{1}{7} = 0.142857 \)
H. 12.34565656

8.NS.1 from illustrativemathematics.org
Assessing Standards - Middle #3

A dollar bill weighs one gram. How many pounds do a million \((10^6)\) dollar bills weigh? \((10^3\) grams is 1 kilogram, and 1 kilogram is 2.205 pounds.)

A dollar bill is 0.0043 inches thick. How many yards high is a pile of \(10^6\) dollar bills?

How would you code this task to the standards?

Describe the degree of alignment, taking into account the context.

Could this task be improved to better align, or improved in other ways?

What questions did you ask while examining the tasks to determine quality and/or alignment?

Source: http://map.mathshell.org.uk/materials/tasks.php

8.EE from Shell Center

Working with rational exponents

Also, some measurement aspects
Assessing Standards - Middle #4

Sam wants to take his MP3 player and his video game player on a car trip. An hour before they plan to leave, he realizes that he forgot to charge the batteries last night. At that point, he plugged in both devices so they can charge as long as possible before they leave. Sam knows that his MP3 player has 40% of its battery life left and that the battery charges by an additional 12 percentage points every 15 minutes. His video game player is new, so Sam doesn’t know how fast it is charging but he recorded the battery charge for the first 30 minutes after he plugged it in.

<table>
<thead>
<tr>
<th>Time (charging minutes)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Video game player battery charge (%)</td>
<td>20</td>
<td>32</td>
<td>44</td>
<td>56</td>
</tr>
</tbody>
</table>

If Sam’s family leaves as planned, what percent of the battery will be charged for each of the two devices when they leave?

How many total minutes would it take to charge the battery 100% on both devices?

Describe the degree of alignment, taking into account the context.

Could this task be improved to better align, or improved in other ways?

What questions did you ask while examining the tasks to determine quality and/or alignment?

Source: www.illustrativemathematics.org

8.F.2 from illustrativemathematics.org
Suppose $P$ and $Q$ give the sizes of two different animal populations, where $Q > P$.

In a. - d., say which of the given pair of expression is larger. Briefly explain your reasoning in terms of the two populations.

a. $P + Q$ and $2P$

b. $\frac{P}{P+Q}$ and $\frac{P+Q}{2}$

c. $(Q-P)/2$ and $Q-P/2$

d. $P + 50t$ and $Q + 50t$

How would you code this task to the standards?

Describe the degree of alignment, taking into account the context.

Could this task be improved to better align, or improved in other ways?

What questions did you ask while examining the tasks to determine quality and/or alignment?

Source: www.illustrativemathematics.org
Assessing Standards - Secondary #2

Given a function \( f \), is the statement

\[ f(x + h) = f(x) + f(h) \]

true for any two numbers \( x \) and \( h \)? If so, prove it. If not, find a function for which the statement is true and a function for which the statement is false.

How would you code this task to the standards?

Describe the degree of alignment, taking into account the context.

Could this task be improved to better align, or improved in other ways?

What questions did you ask while examining the tasks to determine quality and/or alignment?

Source: www.illustrativemathematics.org

F-IF.2 from illustrativemathematics.org
Assessing Standards - Secondary #3

How many solutions are there to the following equation?

\[ 1 - |x| = 2 \]

How many solutions can the equation \( 1 - |x| = c \) have, for different values of \( c \)?

How would you code this task to the standards?

Describe the degree of alignment, taking into account the context.

Could this task be improved to better align, or improved in other ways?

What questions did you ask while examining the tasks to determine quality and/or alignment?

N-Q.1 from illustrativemathematics.org

Sidenote... Pennies after 1982 are only plated with copper (they are 97.5% zinc). Before 1982, they were 95% copper. 😊
Comparing fractions starts in 3.NF.3
Decimal notation of fractions starts in 4.NF
Conceptual Understanding - #2

Check the box if the given expression is a polynomial.

☐ $(3 - t^2)(4 - t^2)(5 - t^2)$
☐ $\pi r^2$
☐ $\frac{2}{3}$
☐ $\frac{1}{2x}$
☐ $1 - t^2$
☐ $\sqrt{2}$

How would you code this task to the standards?

Describe why the task does or does not assess conceptual understanding of the pure mathematics in the standard(s) you identified.

What is the role of technology in assessing conceptual understanding?

Source: Hypothetical example by Jason Zimba

A-APR.1
Conceptual Understanding - #3

Decide whether each of the following numbers is rational or irrational. Explain how you know.

A. $0.33\overline{3}$
B. $\sqrt{4}$
C. $\sqrt{2} = 1.414213\ldots$
D. $1.414213$
E. $\pi = 3.141592\ldots$
F. $11$
G. $\frac{1}{7} = 0.142857\ldots$
H. $12.34565656$

How would you code this task to the standards?

Describe why the task does or does not assess conceptual understanding of the pure mathematics in the standard(s) you identified.

What is the role of technology in assessing conceptual understanding?

Source: www.illustrativemathematics.org
Conceptual Understanding - #4

For each picture, explain how you can see that half of the square is shaded.

How would you code this task to the standards?

Describe why the task does or does not assess conceptual understanding of the pure mathematics in the standard(s) you identified.

What is the role of technology in assessing conceptual understanding?

Source: www.illustrativemathematics.org

3.MD.7d, 3.G.2 from illustrativemathematics.org